ANSWER KEY: ONE-SAMPLE T-TEST FOR POPULATION MEAN

This answer key provides solutions to the corresponding activity sheet.

One-Sample *t*-Test for Population Mean

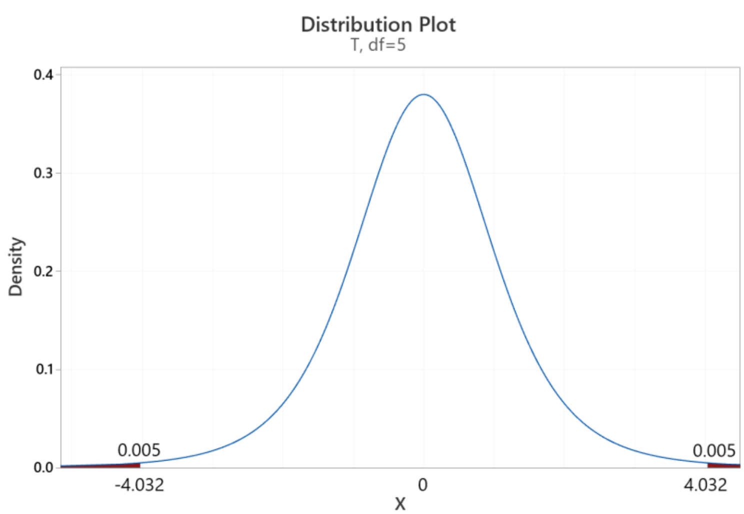
The data for these exercises are in the Minitab file ***OneSampleTTest_PopMean_Activity.mtw***.

Exercise 1

For the following multiple-choice and true/false problems, choose the correct answer.

(a) True or False. The t values that corresponds to $\alpha = 0.01$ in both tails of the t distribution with 5 degrees of freedom are $\pm t_{0.005,5} = \pm 4.032$.

Solution: True. Since we want 0.01 in both tails of the distribution, then we only want 0.005 in one tail. We can use Minitab's **Graph > Probability Distribution Plot** to determine these critical values.



The instructions for determining these values in Minitab are:

Minitab desktop (20 or higher)

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **t**. Type the *df* value *5* in the **Degrees of Freedom** textbox.
4 Click the **Shaded Area** tab. Under **Define Shaded Area By**, choose **Probability**.
5 Click **Both Tails**, since we want the *t* value corresponding to α = *0.01* in both tails. In **Probability**, type the probability value *0.01*.
6 Click **OK**.

Minitab web app

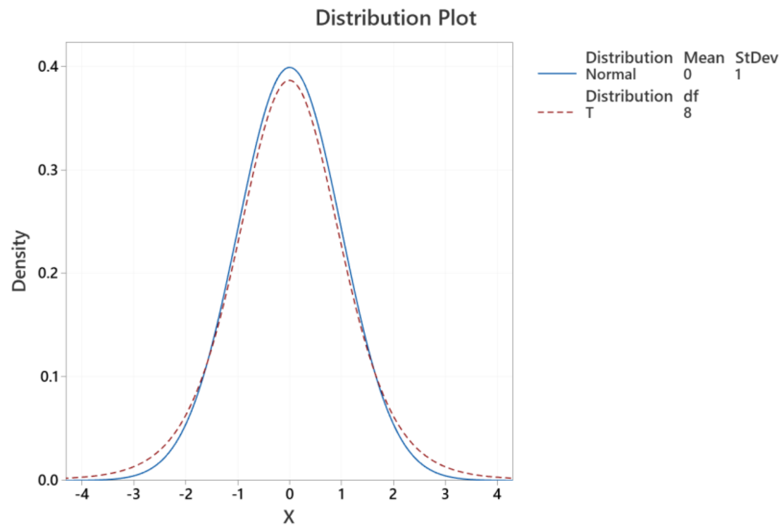
1 Choose **Graph > Probability Distribution Plot**.
2 Under **One Curve**, choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **t**. Type the *df* value *5* in the **Degrees of Freedom** textbox.
4 Click **Options**. Under **Define Shaded Area By**, choose **A specified probability**.
5 Click **Equal Tails**, since we want the *t* value corresponding to α = *0.01* in both tails. In **Probability**, type the probability value *0.01*.
6 Click **OK**.

As displayed in the *t* distribution plot using this procedure, the *t* critical values are $\pm t_{0.005,5}$ = ±4.032

(b) True or False. The *t* distribution has "heavier" tails than the normal distribution, especially for small values of the sample size *n*. That is, there is more area in the tails of a *t* distribution for $n < 15$ than there is for the normal distribution.

Solution: True. This fact is in the ***One-Sample t-Test for Population Mean*** lesson. Recall that the *t* distribution "approaches" a normal distribution as *n* increases.

Using Minitab's **Graph > Probability Distribution Plot > Two Distributions** (in Minitab desktop) or **Graph > Probability Distribution Plot > Compare Two Distributions** (in Minitab web app), we can create multiple graphs in the same plot. I created a normal distribution with mean 0 and standard deviation 1 and *t* distribution with 8 degrees of freedom.

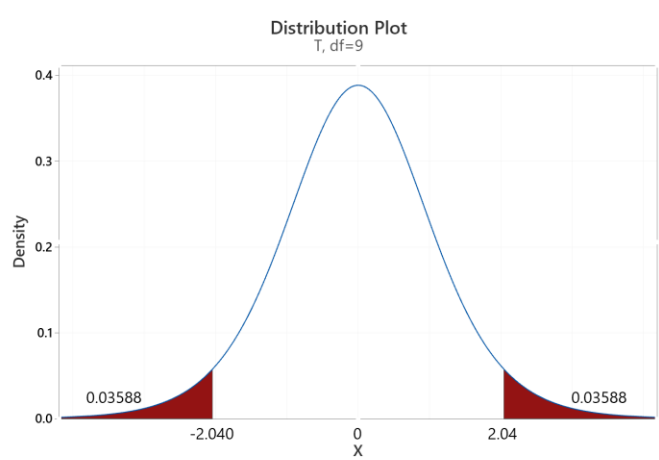


(c) True or False. In a test of H_0: $\mu = 8$ versus H_a: $\mu \neq 8$, a one-sample *t*-test results in a *p*-value of 0.034. Thus, if we create a 95% two-sided confidence interval for μ with the data used to perform the hypothesis test, the interval will include $\mu = 8$.

Solution: False. Since the *p*-value of 0.034 is less than $\alpha = 0.05$, then the two-sided 95% confidence interval for μ will not contain 8.

(d) True or False. You are conducting the hypothesis test H_0: $\mu = 12$ versus H_a: $\mu \neq 12$ for a normally distributed population in which σ is unknown. For a sample size of $n = 10$, you obtain the standardized test statistic $t = 2.04$. The *p*-value for this hypothesis test is approximately 0.07 (rounded correctly to 2 decimal places).

Solution: True. I determined the *p*-value in Minitab using **Graph > Probability Distribution Plot > t** distribution with $df = 9$ and test statistic *2.04*. The corresponding *p*-value is approximately $0.03588 \cdot 2 \cong 0.07176$.



(e) A horticulturist wants to estimate the mean growth of seedlings in a large timber plot planted last year. The growth of seedlings in these conditions is known to be normally distributed. A random sample of n = 10 seedlings is selected and the one-year growth for each is measured. The sample results are: $\bar{x}$ = 5.62 cm and s = 2.50 cm. The 90% two-sided confidence interval for the mean growth μ is:

A. [5.16, 6.08] cm
B. [4.17, 7.07] cm
C. [3.12, 8.12] cm
D. [4.98, 6.26] cm
E. [4.32, 6.92] cm

Solution: I'm assuming the 10 seedlings are a simple random sample of all seedlings in this timber plot. Since their growth is normally distributed, the sample size n is small, and the population variance is unknown, then a t distribution is appropriate for determining a confidence interval for μ.

$$5.62 \pm 1.833 \cdot \frac{2.50}{\sqrt{10}} \cong [\mathbf{4.17, 7.07}]\ \boldsymbol{cm}$$

To compute this interval using Minitab:

1. Choose **Stat > Basic Statistics > 1-Sample t**.
2. From the drop-down list, select **Summarized data**.
3. In **Sample size**, enter *10*.
4. In **Sample mean**, enter *5.62*.
5. In **Standard deviation**, enter *2.50*.
6. Click **Options**. In **Confidence level**, enter *90*.
7. Click **OK**.

Minitab yields the following 90% confidence interval:

Descriptive Statistics

N	Mean	StDev	SE Mean	90% CI for μ
10	5.620	2.500	0.791	(4.171, 7.069)

μ: population mean of Sample

(f) True or False. The average length of time that an engine is allowed to remain running after it is cranked ("run time") is a major factor influencing the longevity of the engine. A new model car is tuned such that it will experience optimal engine life if the true mean run time is 12.5 minutes. An owner of this new model car records her run time for all the trips that she drives during the third month of ownership. Below is a random sample of 12 of her run times.

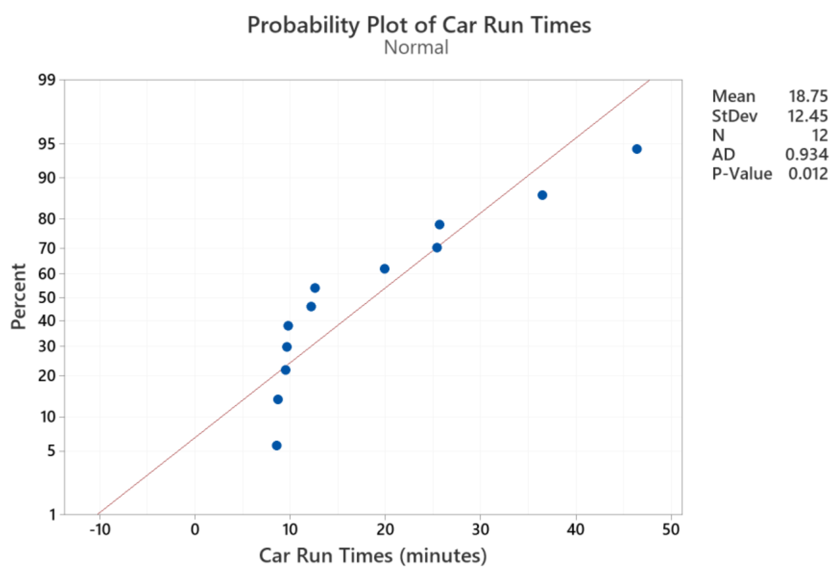
12.6	8.6	12.2	25.4	46.4	36.5	9.8	9.7	19.9	9.5	8.7	25.7

In order to perform a hypothesis test to determine if the true mean run time μ for her car is 12.5 minutes, she should use a one-sample *t*-test with *df* = 11.

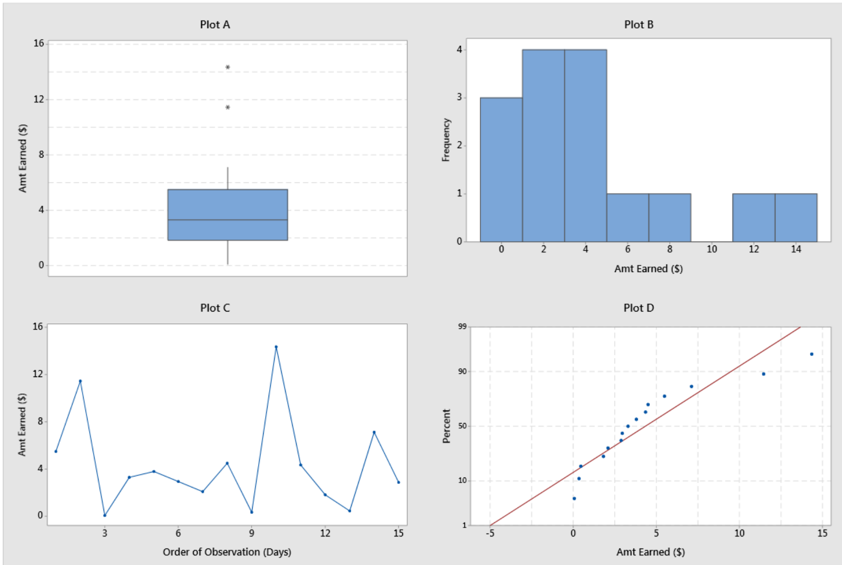
Solution: False. Because *n* is "small" and we are not told that the data is from a normally distributed population, we need to use a normality test to determine if the sample data appears to be from a normal distribution. To check normality, we use Minitab's Anderson-Darling normality test. The Minitab instructions for producing an AD test statistic and probability plot are:

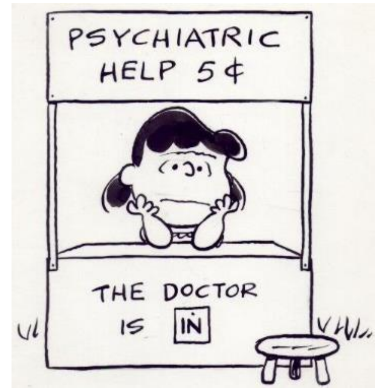
1. Choose **Stat > Basic Statistics > Normality Test**.
2. In the **Variable** text box, select the column containing the sample data of interest.
3. Under **Test for Normality**, make sure **Anderson-Darling** is selected.
4. Click **OK**.

Minitab returns the following graphic along with the AD test statistic and its *p*-value. Since the *p*-value is less than α = 0.05, the sample data suggests that it is not from a normal distribution. Without knowing that the data is from a normally distribution population, we cannot perform a one-sample *t*-test on the population mean. Seek your nearest statistician for help with how to perform a hypothesis test in a case like this.



(g) True or False. As an entrepreneur, Lucy sets up a Psychiatric stand instead of the traditional Lemonade stand. She decides to collect some data to determine if, on average, she makes more than $3.75 per day from her stand. She takes a systematic random sample of 15 days (ordered by day) over the summer and records the amount she makes each day. The correct test for her to perform to address her hypothesis is a **one-sample *t*-test**. Below is a summary of the data.





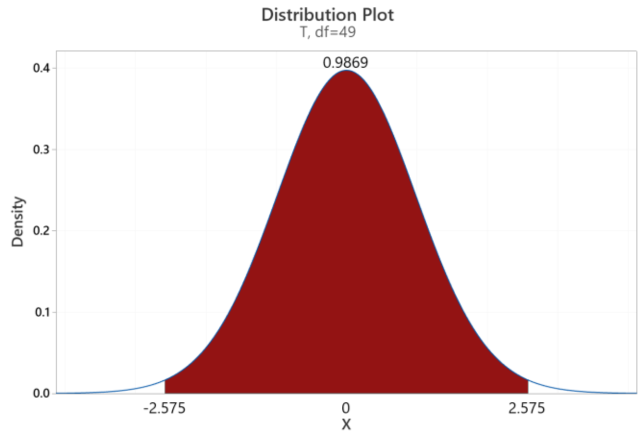
Solution: False. Based on the probability plot (and reinforced by the shape of the boxplot and histogram), the data is not normally distributed. Therefore, the *t* distribution cannot be relied upon to obtain a valid *p*-value.

(h) Suppose we are constructing a two-sided confidence interval for the true population mean by using a simple random sample with sample size n = 50. What is the confidence level associated with the following confidence interval for μ?

$$\bar{x} + 2.575 \cdot s/\sqrt{50}$$

A. 99.5%
B. 99.35%
C. 99%
D. 98.7%
E. 98%

Solution: D. Since n is large, then we can assume that the sampling distribution of the mean is normal according to the Central Limit Theorem (CLT). Since the population standard deviation is unknown, we use a *t* distribution with 49 degrees of freedom. The appropriate *t* values can be obtained by using Minitab's **Graph > Probability Distribution Plot > View Probability > *t*** Distribution with **49** Degrees of freedom.



(i) An operations research analyst for a hotel agency has been asked to develop a fairly accurate estimate of the true mean "check-in" time for customers arriving in the hotel's lobby at noon. The estimate will be used to determine the number of front desk employees required so that a customer's wait time is "reasonable." Suppose the analyst randomly samples 16 customers' check-in times over the next month. She finds that their average wait time is 4.2 minutes with a standard deviation of 1.4 minutes. Which of the following is the correct two-sided 90% confidence interval for the true mean check-in time μ.

A. (3.586, 4.814) minutes
B. (3.454, 4.946) minutes
C. (3.514, 4.886) minutes
D. (3.624, 4.776) minutes
E. We don't have enough information to construct this confidence interval.

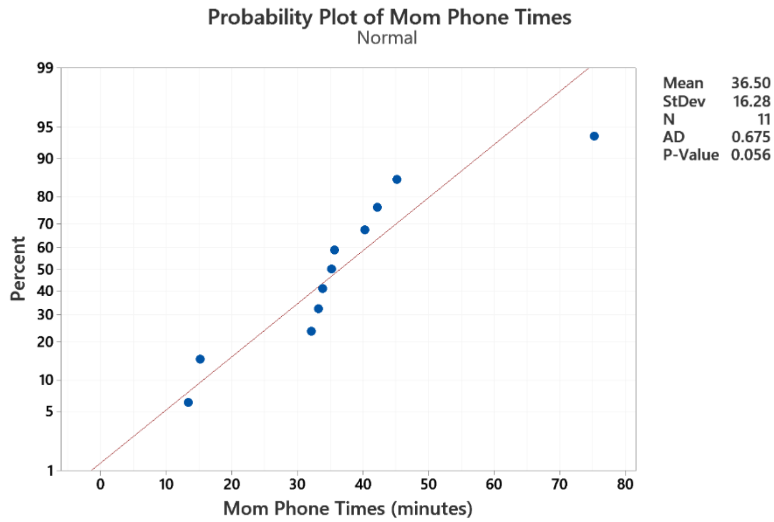
Solution: E. Since the sample size is small, and we don't know the population standard deviation or if the data is from a normal distribution, we can't construct an approximate confidence interval in good faith.

(j) True or False. I talk to my mom on the phone every night. I randomly selected 11 dates (in minutes) up to July 4, 2020 and recorded the phone time usage with my mom. Here is the data:

13.34 35.23 45.12 32.14 33.82 75.23 15.22 42.22 35.68 40.24 33.21

The data is normally distributed at level of significance $\alpha = 0.05$.

Solution: True … but just "barely" at the $\alpha = 0.05$ level. This is real data. I typically talk to my mom around 35 minutes each night. Sometimes I don't get a chance to call until after 10:30 p.m., and the times are smaller (e.g. less than 15 minutes). Sometimes there is something she really needs to talk about, and we talk for a longer time (e.g. more than one hour). To determine if the data is normally distributed, use Minitab's **Stat > Basic Statistics > Normality Test.** The corresponding plot shows that the *p*-value is approximately 0.056. Although it's questionable whether the data is normally distributed at α = 0.05, especially knowing the information provided about our phone conversation times, our data doesn't provide the necessary evidence to reject the null hypothesis of normality. Also, the normality plot doesn't appear to agree with what the "flat pencil test" would suggest, which is non-normality.



(k) True or False. In conducting the hypothesis test H_0: $\mu = 8$ versus H_a: $\mu \neq 8$, the corresponding *p*-value is 0.44. If we construct a 95% two-sided confidence interval for μ using the exact same data, the value 8 would not be included in this 95% confidence interval.

A. True
B. False
C. We do not have enough information to determine this.

Solution: False. At α = 0.05, we would not reject H_0 with a corresponding value of *p*-value 0.44. By not rejecting H_0: $\mu = 8$, we are saying that we don't have the evidence to refute $\mu = 8$. Thus, a 95% *would* contain $\mu = 8$ as a possibility as the true mean of the population.

(l) Suppose you perform the following hypothesis test on the true mean GPA at College A: H_0: $\mu = 3.1$ versus H_a: $\mu > 3.1$. You obtain a *p*-value for the hypothesis test. Which of the following statements is correct regarding the *p*-value?

A. Under the assumption that H_0 is true, an extremely small *p*-value is consistent with the sample mean $\bar{x}$ greatly differing from null mean ($\mu = 3.1$).
B. The *p*-value measures the probability that the alternative hypothesis is true.
C. The *p*-value measures the probability that the null hypothesis is true.
D. The larger the *p*-value, the stronger the evidence against the null hypothesis.
E. A large *p*-value indicates that the data supports with the alternative hypothesis.

Solution: A. If H_0 is true and the true mean GPA at College A is 3.1, then a random sample of data from the population with mean $\mu = 3.1$ should have a sample mean "fairly close" to 3.1. For example, if $\mu = 3.1$ and the population standard deviation is $\sigma = 0.05$, then obtaining a sample mean of 3.6 given a random sample of size $n = 100$ would be practically impossible. The standardized test statistic for this example is $t_0 = 100$. Basically, we're not going to get a sample mean of $\bar{x} = 3.6$ under these conditions ($\mu = 3.1$, $\sigma = 0.05$, $n = 100$).

(m) Suppose a company boasts that it has new fertilizer that yields *at least* 2 tons (per year) of a certain crop over 50 acres. Twenty farmers (whose 50 acres usually yield 2 tons) are randomly selected to try the new fertilizer on their crops.

What are the null hypothesis, alternative hypothesis, test statistic and conclusion for testing the company's claim at the $\alpha = 0.05$ significance level? Use the Minitab output below and assume the population (crop growth) is approximately normally distributed.

Descriptive Statistics

N	Mean	StDev	SE Mean
20	1.9800	0.3200	0.0716

μ: population mean of crop growth

A. H_0: $\mu = 2$ versus H_a: $\mu > 2$, $t_0 = -0.28$. The company does not have enough evidence to reject the null hypothesis at $\alpha = 0.05$. Thus, their sample data *does not support* the new fertilizer yielding a larger crop.

B. H_0: $\mu = 2$ versus H_a: $\mu < 2$, $t_0 = -0.28$. The company does not have enough evidence to reject the null hypothesis at $\alpha = 0.05$. Thus, their sample data *does support* the new fertilizer yielding a larger crop.

C. H_0: $\mu = 2$ versus H_a: $\mu > 2$, $t_0 = -0.28$. The company does have enough evidence to reject the null hypothesis at $\alpha = 0.05$. Thus, their sample data *does support* that the new fertilizer is yielding a larger crop.

D. H_0: $\mu = 2$ versus H_a: $\mu < 2$, $z_0 = -0.28$. The company does not have enough evidence to reject the null hypothesis at $\alpha = 0.05$. Thus, their sample data *does not support* the new fertilizer yielding a larger crop.

E. H_0: $\mu = 2$ versus H_a: $\mu \neq 2$, $z_0 = -1.25$. The company does not have enough evidence to reject the null hypothesis at $\alpha = 0.05$. Thus, their sample data *does not support* the new fertilizer yielding a larger crop.

Solution: A. We are told to assume that the population (crop growth) is approximately normally distributed. We do not know the population standard deviation. The correct distribution to model the sample means is a *t* distribution with *df* = 19. The correct test statistic is:

$$t_0 = \frac{1.98 - 2}{\frac{0.32}{\sqrt{20}}} \cong -0.28.$$

The p-value that accompanies this test statistic is $p \approx 0.609$. Since the p-value is greater than $\alpha = 0.05$, the sample data doesn't provide enough evidence to reject H_0. The company doesn't have evidence to support that their new fertilizer yields a larger crop growth.

(n) What is the p-value associated with the following hypothesis test with sample data as shown below?

$$H_0: \mu = 28 \text{ vs } H_a: \mu \neq 28$$

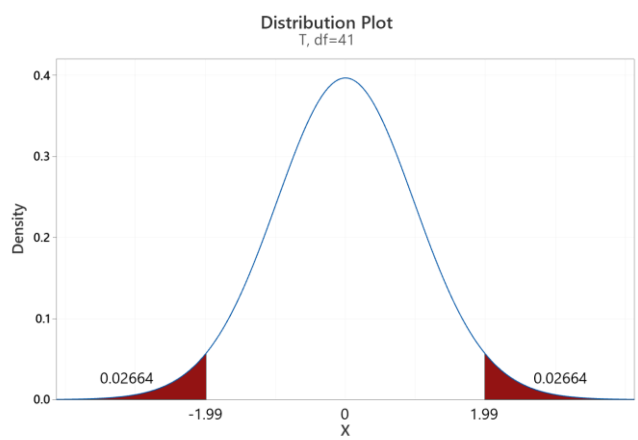
Descriptive Statistics

N	Mean	StDev	SE Mean
42	28.500	1.630	0.252

μ: population mean

A. There is not enough information to determine an answer
B. 0.000
C. 0.054
D. 0.023
E. 0.027
F. 0.047

Solution: C. Since n is large, and we don't know the population standard deviation, then the right distribution to model this data is a t distribution with $df = 41$. Since it is a "not equal to" alternative hypothesis, the p-value includes the area in both tails of the t distribution associated with the standardized test statistic $t_0 \approx 1.99$.



(o) We are performing a one-sample t-test on a population mean μ in Minitab. Assume that we are told that the population is normally distributed. Here are the null and alternative hypotheses:

$$H_0: \mu = 75 \text{ vs } H_a: \mu \neq 75$$

Suppose we only obtain the following Minitab results. Use these results to answer the two questions below.

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
20	70.85	6.56	1.47	(66.65, 75.05)

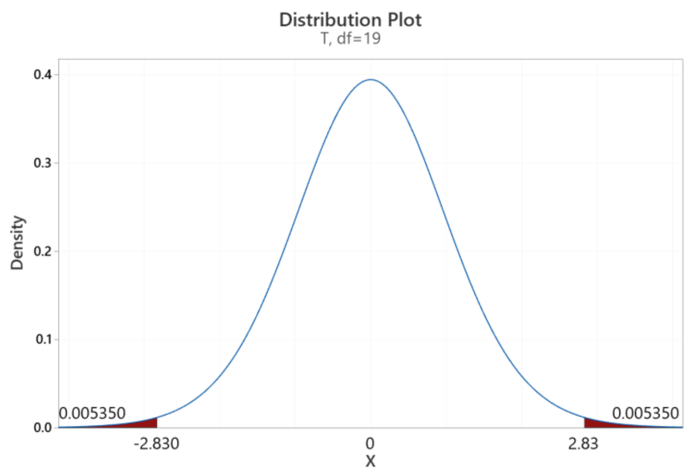
μ: population mean of home prices in a certain city

i. **True or False**. H_0 should be rejected at level of significance α = 0.05.

Solution: True. It's easier to start with part (i) for this problem. Since μ = 75 is inside of the two-sided 99% confidence interval for μ, then we would not reject H_0: μ = 75 at α = 0.01. This mean the *p*-value for this hypothesis test is greater than 0.01. But how much greater is the *p*-value? If it's between 0.01 and 0.05, then we would reject H_0 at α = 0.05. If it's larger than 0.05, then we would not reject H_0. Let's determine the *p*-value by first calculating the test statistic for this hypothesis test:

$$t_0 = \frac{70.85 - 75}{\frac{6.56}{\sqrt{20}}} \cong -2.83$$

The *p*-value for t_0 for a "not equal to" alternative hypothesis is ~0.0107. So, we would reject H_0 at α = 0.05.



ii. **True or False**. H_0 should be rejected at level of significance α = 0.01.

Solution: False. The rationale for this answer is discussed in the solution to part (i).

Exercise 2 based on the movie "Beauty and the Beast"

To win first prize at the inventor's fair, Maurice designs a machine that improves wood chopping. He is interested in establishing that, on average, his machine can chop *more than* 420 lbs. of wood per hour. He conducts a study in which the machine is run for 20 one-hour intervals. At the end of each hour, he weighs the amount of wood chopped.

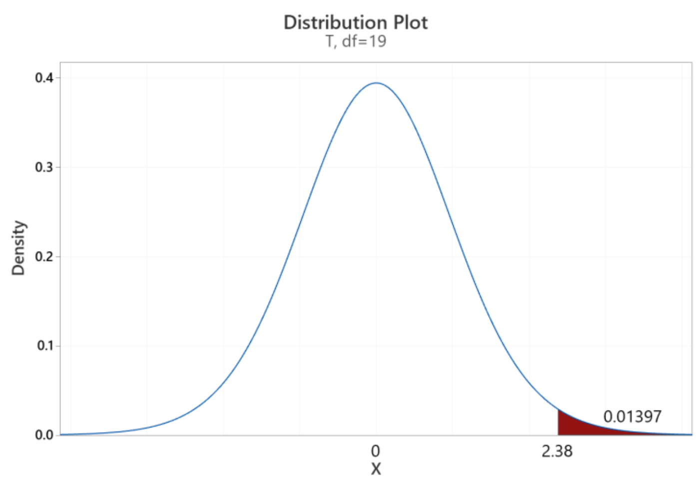
(a) State the null and alternative hypothesis appropriate for addressing Maurice's question of interest. Be sure to define symbols used to represent the parameter.

Solution: The parameter of interest, μ, is the mean wood chopping amount (in lbs.) for the machine that he has invented.

$$H_0: \mu = 420 \quad \text{vs.} \quad H_a: \mu > 420$$

(b) Using the data that he collected, Maurice computes a test statistic of $t_0 = 2.38$. Assuming all necessary assumptions are reasonable for using a *t* distribution to model his data (e.g. normality), compute the *p*-value that accompanies Maurice's output.

Solution: Assuming that the *t* distribution is the correct model and the degrees of freedom are 19, the correct *p*-value is approximately 0.014. Note that we are only interested in the area in the right tail of the *t* distribution since the alternative hypothesis is "greater than."



(c) Assuming that all necessary assumptions are reasonable, what conclusions can Maurice make at the α = 0.05 significance level? Be sure to state the conclusions in the context of the problem.

Solution: Since the *p*-value is small, we reject the null hypothesis and conclude that there is evidence that his machine can chop more than 420 lbs. of wood per hour.

Exercise 3 based on the movie "Beauty and the Beast"

As a gift to Belle, the Beast presented her with a library full of books. Belle is determined to read every book in the library. Whether she can accomplish her goal partly depends on the length of the books. Let μ represent the average length (number of pages) of a book in the library. Belle is interested in determining if:

$$H_0: \mu = 500 \text{ vs. } H_a: \mu < 500$$

She takes a random sample of 50 books and records the number of pages contained in each. Below is a partial printout of the Minitab output corresponding to her analysis.

N	Mean	t
50	476.4	-1.04

μ: population mean of the number of book pages

(a) After looking at the results from Belle's analysis, Beast states that "there is insufficient evidence, at the $\alpha = 0.05$ level, to conclude that there are less than 500 pages in a book, on average." Is his statement justified? Explain.

Solution: Yes. Resist the temptation to state the acceptance of the null hypothesis. If Beast told her that "there are at least 500 pages in a book, on average," this would be stating the acceptance of the null hypothesis. Hypothesis test conclusions are based on rejecting or not rejecting H_0. We never say that the null hypothesis is true. We can say that "the data is consistent with not being able to reject H_0."

(b) Which of the following **is** an appropriate interpretation of the *p*-value for this test?

A. The probability that the alternative hypothesis is true is 0.151.
B. The probability that the null hypothesis is true is 0.151.
C. In repeated sampling, the probability of observing a test statistic less than or equal to -1.04 if the alternative hypothesis is true is 0.151.
D. In repeated sampling, the probability of observing a test statistic less than or equal to -1.04 if the null hypothesis is true is 0.151.

Solution: D. A *p*-value is associated with repeated sampling under the assumption that the null hypothesis is true.

(c) Which of the following would be an appropriate two-sided confidence interval to assess the hypothesis $H_0: \mu = 500$ vs. $H_a: \mu < 500$ at the *$\alpha = 0.02$ significance level*?

96% *z*-interval	98% *z*-interval	99% *z*-interval
96% *t*-interval	98% *t*-interval	99% *t*-interval

Solution: 96% *t*-interval. First, $n = 50$ allows us to assume the distribution of the sample mean is normal. The population standard deviation is unknown. Since she has a 1-sided "less than" alternative hypothesis, we need to construct a $100(1 - 2\alpha)\%$ confidence interval. That is, since we are deciding at the α = 0.02 level, we need 0.02 are in *both* tails of the *t* distribution. With a total of 0.04 outside of the two-sided confidence interval, we have 0.96 within it.

(d) Suppose she repeats the study increasing the sample size to n = 100. *Holding all else fixed*, what effect would this have on the reported p-value of 0.151?

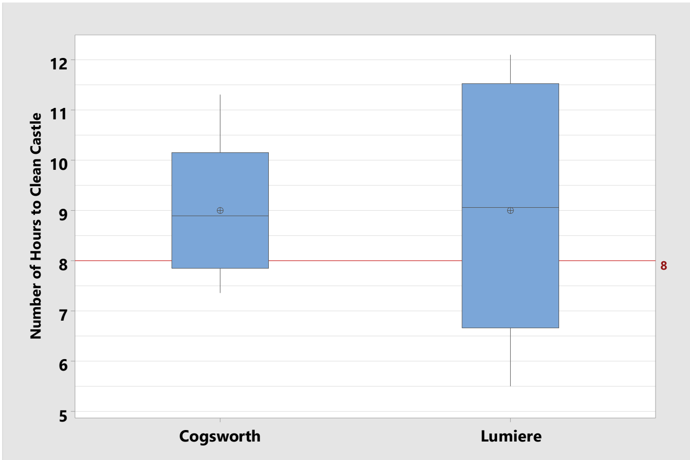
A. The *p*-value will decrease.
B. The p-value will remain the same.
C. The p-value will increase.
D. It is not possible to say how the p-value will change.

Solution: A. Increasing the sample size will increase the standardized *t* test statistic *in absolute value*. This means that t_0 will become "more negative."

$$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Therefore, the test statistic will be further out in the left tail of the *t* distribution, and the corresponding p-value will decrease.

(c) Lumiere and Cogsworth each are responsible for overseeing the castle. Suppose they are interested in testing whether Plumette, a feather duster, requires *more than* 8 hours, on average, to clean the entire castle. Cogsworth and Lumiere decide to collect separate samples, each taking a sample of size 20. A side-by-side boxplot of their data is shown below. Which character will obtain a *larger* test statistic for assessing the question of interest? Explain.



Solution: The alternative hypothesis is "greater than," and both Cogsworth's and Lumiere's sample means are larger than 8 as shown in the diagram above. Both of their data sets will correspond to positive standardized test statistics. Since Cogsworth's data has a smaller sample standard deviation, his test statistic will be larger.

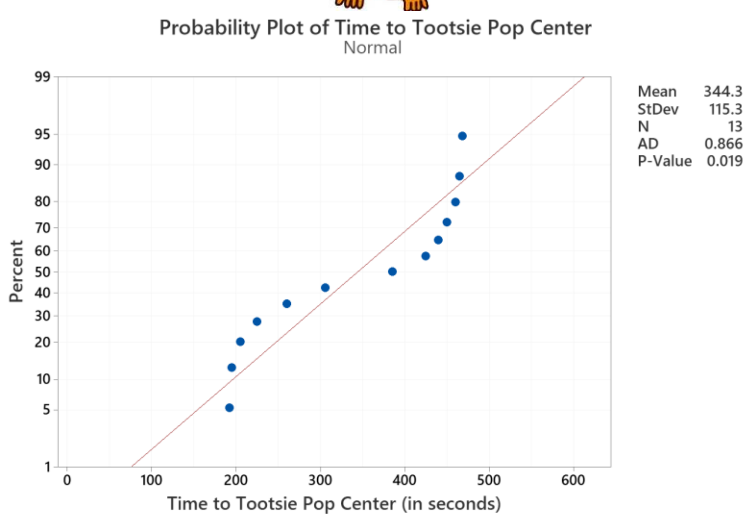
Exercise 4: Tootsie Pops

Below are data results for a random sample of n = 13 students in an Engineering Statistics class who were given Tootsie Pop suckers and asked to record how long it took (in seconds) until they first tasted the inside tootsie roll center.

205	440	425	192	195	225	260	465	306	450	385	460	468

Using the appropriate method, determine a two-sided 90% confidence interval for the true mean amount of time to get to the center of a Tootsie Pop for these students. Report your final confidence interval values correct to 2 decimal places.

Solution: We do not have enough information to construct this confidence interval. Since n is small, then we cannot say that the distribution of the sample means is normally distributed. This means that we need to check normality. The AD normality test yields of p-value of 0.019. Since this p-value is less than $\alpha = 0.05$, we reject the null hypothesis of the normality test that states the data is from a normal distribution. Without knowing that the data is from a normally distribution population, we cannot construct a one-sample t-test for the population mean. Seek your nearest statistician for help with how to construct a confidence interval in a case like this.



Exercise 5: Restaurant Clean-up Times

A restaurant employee (with a statistics background) at “Sushi Yummy” has been asked to provide a 99% two-sided confidence interval the true mean time to clean off a restaurant table and get it prepared for the next customer on a busy Saturday night. This knowledge will help the restaurant owners know how much time they should schedule between reservations. Suppose the employee randomly samples table clean up times for n = 40 tables on busy Saturday nights and obtains a sample mean clean-up time of $\bar{x} = 4.2$ minutes with a sample standard deviation of s = 0.8 minutes.

The sample data is used to construct the following two-sided confidence interval for the true mean clean up time μ:

(3.962, 4.438) minutes

Determine the approximate confidence level (e.g., 75%, 88%, 95%) of this confidence interval correct to 2 decimal places.

Solution: Since n is large and we don't know the population standard deviation, then the correct distribution to model this data is a t distribution with df = 39. We can determine the area below 3.962 and above 4.438 by determining the standardized t values associated with each.

$$t_{lower} \cong \frac{3.962 - 4.2}{\frac{0.8}{\sqrt{40}}} \cong -1.881555$$

$$t_{upper} \cong \frac{4.438 - 4.2}{\frac{0.8}{\sqrt{40}}} \cong 1.881555$$

Since the t distribution is symmetric, the area below -1.881555 and the area above 1.88155 are the same. We just need to determine the area beyond one of them and then double it. Using Minitab's **Graph > Probability Distribution Plot > t** distribution, **degrees of freedom** = 39, we obtain $P(t_{upper} > 1.881555) \approx 0.03369$. Thus, the confidence level is:

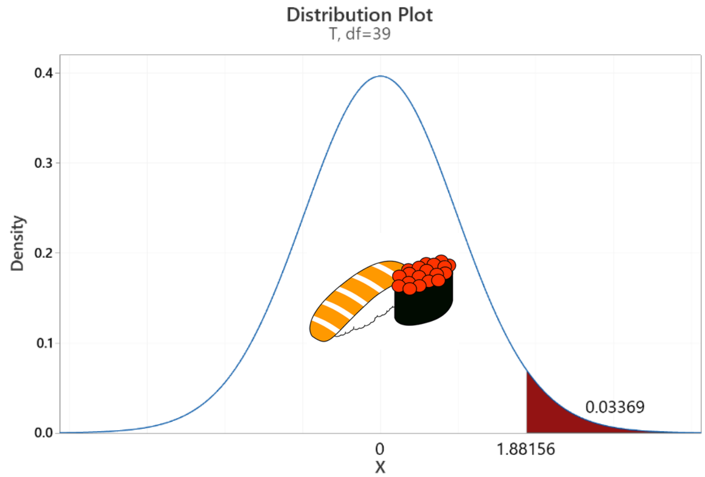
$$100 \cdot (1 - 2 \cdot 0.03369)\% \cong 93.26\%$$

We can check the answer by inserting the sample data in Minitab's **Stat > Basic Statistics > 1-Sample t** and selecting **Options** to enter the value 93.26 in the **Confidence Level** textbox. The resulting 93.26% confidence interval is:

Descriptive Statistics

N	Mean	StDev	SE Mean	93.26% CI for μ
40	4.200	0.800	0.126	(3.962, 4.438)

μ: population mean clean-up time



Exercise 6: Low Tire Pressure

(From the article *Whatever Happened to...Full Service Gas Stations?* at Go Retro.*) The oil crisis of the 1970s marked the beginning of the end for the full-service gas station. Oil companies figured that customers wanted to pump their own gas in exchange for saving a few pennies. Pretty soon, the attendants were no longer needed. Also, the process of getting gas at a full-service station took about 10-15 minutes, which sadly is considered too long in today's high paced, impatient world.

* https://www.goretro.com/2011/07/whatever-happened-tofull-service-gas.html

As a result of gas stations moving from full-service to self-service, a certain gas station in a small town is concerned that many cars are being driven on underinflated tires. This can result in excessive tire wear, and unsafe steering and braking of the car. A tire is seriously underinflated if its tire pressure is more than 10 psi under its recommended level.

The gas station manager selects a random sample of 100 cars at his station and records the cars' front driver's side tire pressure while the customer pumps their gas. He determines that the mean underinflation for this random sample is 10.4 psi with a standard deviation of $s = 4.2$ psi.

(a) Construct a 99% two-sided confidence interval for the mean underinflation μ.

Solution: Since the sample size is large and the population standard deviation is unknown, we use a *t* distribution with *df* = 99 to construct the interval with the sample standard deviation s. The 99% confidence interval for the mean underinflation μ is:

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
100	10.400	4.200	0.420	(9.297, 11.503)

μ: population mean of the underinflation of tires at this gas station

(b) Based on your confidence interval from part (a), would you recommend that the manager issue a report that the mean tire pressure is seriously underinflated? Explain your answer.

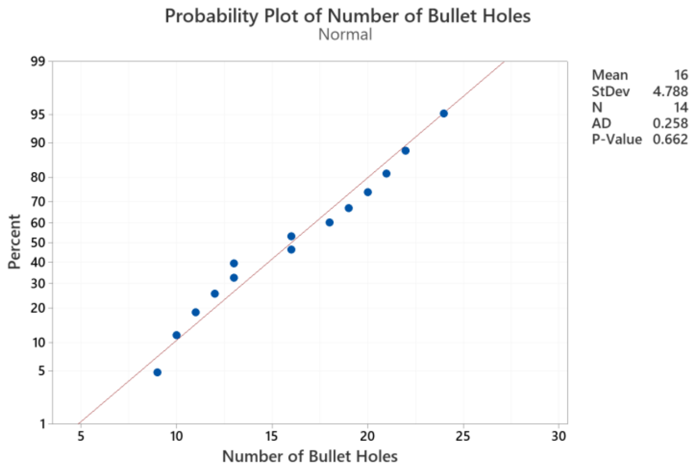
Solution: No. There is a 99% chance that the interval (9.297, 11.503) psi contains the average underinflation μ for cars at this gas station. A tire is only considered seriously underinflated if the tire pressure is less than 10 psi. Fortunately, the manager doesn't need to issue a report to his customers that the mean tire pressure is seriously underinflated because the 99% confidence interval contains 10 psi. Technically, since we just want to consider underinflated rather than overinflated tires, the one-sided 98% CI for μ has the upper bound of 11.503 psi.

Exercise 8: Taking Fire

The World War I Flying Ace (a.k.a. Snoopy) often searches the skies for his enemy, The Red Baron. As a result, his aircraft often sustains damage from shots fired from the Baron. The Ace's commanding officers are interested in estimating the average number of bullet holes in the side of the Ace's aircraft after an encounter with the Baron. For a random sample of 14 flights in which the Ace encountered the Baron, the number of bullet holes in the side of the aircraft is recorded. The data for this problem is in the column "Number of Bullet Holes" in the Minitab worksheet associated with this lesson's activities.

(a) The commanding officers would like to construct a confidence interval for the mean number of bullets; they would like to use a "*t*-interval." Based on the data for this problem, is this the correct interval to construct? Explain.

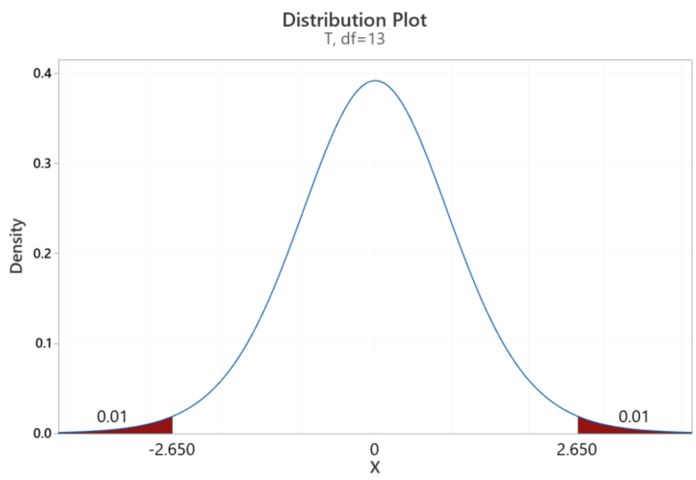
Solution: Since the sample size is small (n = 14), we need to determine if it is reasonable to assume that the underlying population follows a normal distribution. We do this with Minitab's **Stat > Basic Statistics > Normality test**. Since the AD test statistic for the normality test has a p-value greater than 0.05, it is reasonable to assume that the population is normally distributed.



(b) Regardless of what you concluded in part (a), the officers constructed a 98% "t-interval" to estimate the mean and obtained the following interval: (12.61, 19.39) bullet holes. What critical value (*cv*) was used to construct this interval? See the expression below for further clarification on the value *cv* you are determining.

$$\bar{x} \pm \boldsymbol{cv} \cdot \frac{s}{\sqrt{n}} \rightarrow (12.61, 19.39) \text{ bullet holes}$$

Solution: Method 1: Since this value comes from a *t* distribution with *df* = 13, the *cv* can be determined using Minitab's **Graph > Probability Distribution Plot > t** distribution with **Degrees of freedom** 13. Since the confidence interval level is 98% and it's a two-sided interval, then we want 0.01 area in each tail of the *t* distribution. The *cv* is 2.650.



Method 2: Some students may backwards solve for the critical value. Determine the *cv* such that:

$$16 + cv \cdot \frac{4.788}{\sqrt{14}} = 19.39 \rightarrow cv = \mathbf{2.649}$$

(c) Regardless of your answer to part (a), assume the underlying population is normally distributed. The Flying Ace states that he is able to escape from the Red Baron with fewer than 15 bullet holes, on average. Using the interval provided in part (b), is his statement justified? Explain.

Solution: Yes, it is justified because 15 is within the confidence interval.

(d) Regardless of your answer to part (a), assume the underlying population is normally distributed. Which of the following is an appropriate interpretation of the interval stated in part (b)?

A. 98% of flights observed had between 12.61 and 19.39 bullet holes.
B. 98% of all flights have between 12.61 and 19.39 bullet holes.
C. There is a 98% chance that the average number of bullet holes for this sample is between 12.61 and 19.39.
D. There is a 98% chance that the average number of bullet holes in the population is between 12.61 and 19.39.
E. None of the above is a correct interpretation.

Solution: E. We know that if we were to repeat the study there is a 98% chance that the process would result in an interval that captured the parameter.

(e) Suppose the officers were to repeat the study collecting a new random sample of 14 flights and observed a sample standard deviation of 0. Which of the following statements *must* be true?

A. Half of the values observed are above the sample mean.
B. All of the values observed are zero.
C. All of the values observed are equivalent.
D. The values are evenly spaced on both sides of the sample mean.

Solution: C. A standard deviation of 0 implies there is no variability in the data, and all the values are equivalent.